

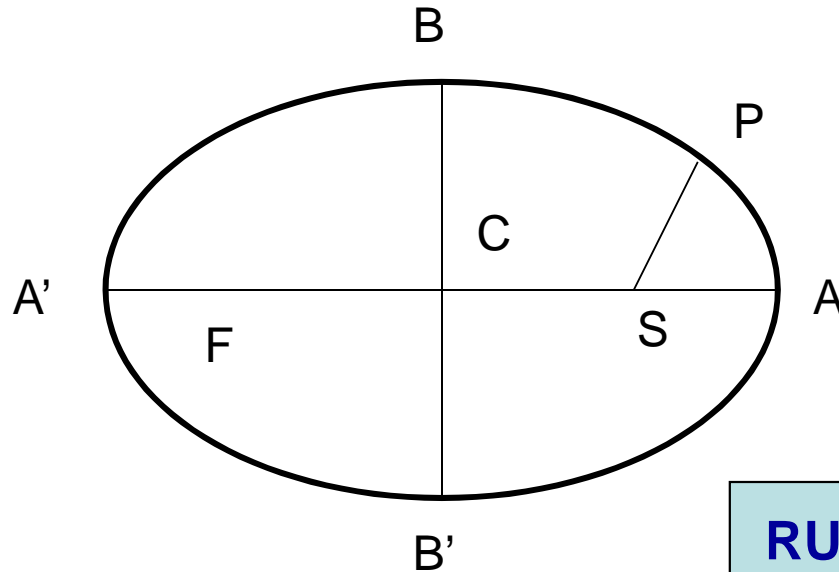
MEKANIKA IPBA

Insih Wilujeng

CELESTIAL MECHANICS: The Two-Body Problem

- **Planetary Orbits** : Kepler's First Law; Kepler's Second Law; Kepler's Third Law
- **Newton's Laws of Motion**
- **Newton's Law of Gravitation**
- **The Principis of Isaac Newton**
- **The two-Body Problem**: Equations of Motion; the solution of two-body problem; the energy integral; the velocity a planet in its orbits; the period of revolution of a planet in its orbit; newton's form of kepler's third law, measuring the mass of a planet
- **The Astronomical Unit**

Kepler's First Law



AA' = major axis, C = center,
CA and CA' = semi-major axes
(a)

BB' = minor axis, CB and CB' =
semi-minor axes (b), e =
eccentricity

SP = the planet's radius vector

**The planet is said to be at
perihelion when it is at A**

**The planet is said to be at
aphelion when it is at A'**

The angle ASP = true anomaly

RUMUS :

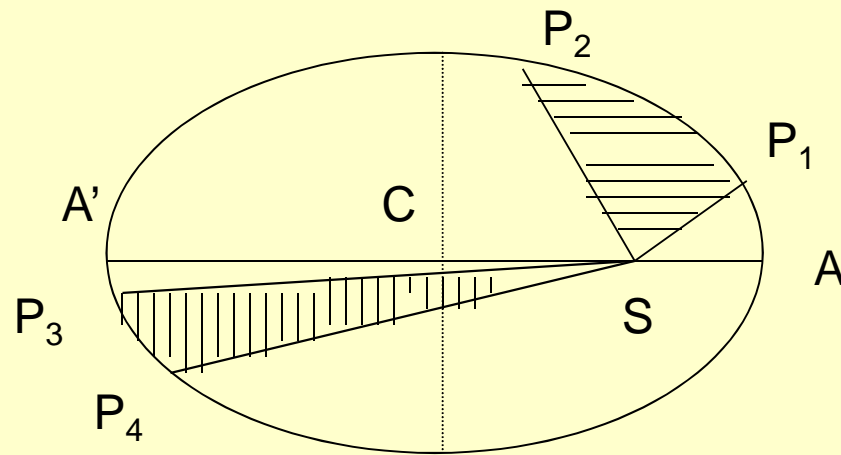
$b^2 = a^2 (1 - e^2)$

$e = CS/CA$

$SA = CA - CS = a - ae = a (1 - e)$

$SA' = CA' + CS = a + ae = a (1 + e)$

Kepler's Second Law



If $\angle P_1SP_2$ expressed in radians, we may write:

$$\text{Area } SP_1P_2 = \frac{1}{2} r_1^2 \theta_1$$

Since $\angle P_1SP_2$ is very small, Also $SP_1 = SP_2 = r_1$

Area SP_1P_2 is given by : $\frac{1}{2} r_1^2 \theta_1$

$$\frac{1}{2} r_1^2 \frac{\theta_1}{t} = \frac{1}{2} r_2^2 \frac{\theta_2}{t} = \text{constant}$$

$$\frac{1}{2} r_1^2 \omega_1 = \frac{1}{2} r_2^2 \omega_2 = \text{constant}$$

Kepler's Third Law

$$\frac{a_1^3}{T_1^2} = \frac{a_2^3}{T_2^2} = \frac{a_3^3}{T_3^2}$$

a = semi-major axis

T = siderial period of a planet

For the Earth, $a_1 = 1$ and $T_1 = 1$, so

$$a^3 = T^2$$

Newton's Laws of Motion

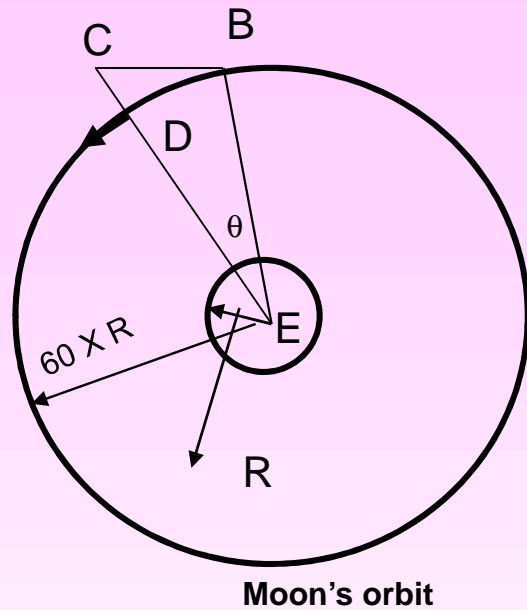
- i. Every body continues in its state of rest or of uniform motion in a straight line except insofar as it is compelled to change that state by an external impressed force
- ii. The rate of change of momentum of the body is proportional to the impressed force and takes place in the direction in which the force acts

$$\frac{d(mv)}{dt} = F$$

$$m \frac{dv}{dt} = F \implies m \cdot a = F$$

- iii. To every action there is an equal and opposite reaction

Newton's Law of Gravitation



Every particle of matter in the universe attracts every other particle matter with a force directly proportional to the product of the masses and in inversely proportional to the square of the distance between them

$$F = G \frac{m_1 m_2}{r^2}$$

BEBERAPA SIFAT PLANET



$$v_c = \sqrt{\frac{GM}{r}}$$

dengan r adalah jarak planet-Matahari

$$v_e = \sqrt{2} v_c$$

Suhu Efektif Planet

Adalah : suatu benda hitam yang menerima dan menyerap (tentu saja memancarkan kembali) sejumlah energi radiasi surya yang sama dengan apa yang dilakukan planet itu

Suhu efektif planet secara kasar dapat disamakan dengan suhu kinetik permukaan planet atau pada lapisan atmosfer planet yang memancarkan kembali sebagian besar energi yang diserap

Rumusan Suhu efektif planet

$$T = \left(\frac{(1-A) \times 1370}{4\sigma r^2} \right)^{1/4}$$

A = Albedo

1370 joule/dt = Solar Constant

$\sigma = 5,672 \times 10^{-8} \text{ watt/m}^2 \cdot \text{K}^4$

Atmosfer Planet

Unsur-unsur pokok atmosfer planet diketahui dengan cara menganalisis spektroskopi terhadap cahaya matahari yang dipantulkan planet ke bumi atau dengan alat ukur pesawat angkasa antar planet

Unsur-unsur pokok atmosfer planet dipengaruhi pula oleh proses terbentuknya planet, misalnya

Planet terrestrial atmosfernya berasal dari gas-gas yang keluar dari batuan kerak bumi

Planet Jovian atmosfernya menunjukkan komposisi asli ketika planet terbentuk

- Kemampuan planet untuk mengikat lapisan atmosfer sangat dipengaruhi oleh massa, jejari dan suhu planet
- Mengapa molekul-molekul gas bagian bawah atmosfer tidak mampu melepaskan diri ke luar angkasa?

Jawabnya adalah, karena molekul-molekul gas bertumbukan dengan molekul-molekul gas di atasnya dan terpantul ke bawah.

- Jika kecepatan rerata molekul-molekul gas kurang dari atau sama dengan seperenam kecepatan lepasnya, maka planet-planet akan mengikat molekul-molekul gas tersebut

$$6\sqrt{\frac{3kT}{m}} \leq \sqrt{\frac{2GM}{r}}$$